

higher alcohols, the carbohydrates, the fats and waxes, the phosphatides, the vegetable proteins, and the animal proteins. The volume concludes in the middle of an article by the editor on the disintegration products of the proteins, characterised by much work of a patient and laborious type. The reader is deserted in the middle of a sentence, with "möglichst" for his last word, but it may be confessed that one can wait for the next issue of the fascinating narrative with more patience than one was able to command in earlier days of reading serial literature.

The articles in this part are of unequal value and exhaustiveness; that on the phosphatides is much too short; very valuable are the three articles, on sugars by B. Tollens, that on glycogen by K. Grube, and especially that on the proteins of the vegetable world by T. B. Osborne, which is a model of what an article on practical methods for the laboratory ought to be.

Such articles as these make the work essential in a biochemical laboratory, but it is a pity that they cannot be purchased as monographs in the particular subjects.

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#### THE LIGHT FROM THE SKY.

*Meteorologische Optik.* By Prof. J. M. Pernter. Section IV. Pp. i-xvii, 559-799. By Felix M. Exner. (Vienna and Leipzig: W. Braumüller, 1910.)

THIS volume is the fourth part of the late Prof. Pernter's work on meteorological optics, a notice of the earlier parts of which appeared in NATURE on April 18, 1907. It was undertaken in March, 1908, by Dr. Exner at Pernter's request, at a time when the latter's condition of health not only made it impossible that he should complete the work himself, but prevented him from giving any material assistance or advice in its preparation. The scope of the section was clearly indicated in the original plan of the work, and as regards the method of presentation Exner has successfully followed that of the preceding sections; but he had no notes to assist him, and the volume must therefore be regarded as Exner's work, except in so far as he has utilised, in some portions, Pernter's previously published papers.

In pursuance of the general scheme outlined in the previous notice above referred to, this fourth section is devoted to the discussion of those phenomena which are due to the action of the minute particles of all kinds which are always present in the atmosphere, among which must be included the gaseous molecules themselves. Thus the first two chapters deal with the colour and polarisation of sky-light, the third treats of the loss of light in passing through the atmosphere and the general brightness of daylight, while in the fourth chapter is given a brief account of what are called the phenomena of twilight, the optical effects associated with sunrise and sunset.

Of all the many interesting problems the discussion of which falls under the head of meteorological optics, that of the colour of the sky, with the associated questions as to the polarisation of sky-light, its intensity and composition, and the effects of atmospheric absorption, is perhaps the most fascinating. Less

striking only because not exceptional, in the sense in which this adjective applies to the appearances dealt with in the earlier sections, even for the unscientific observer the everyday recurrence of the phenomena fails to diminish their appeal to his artistic sense and imagination. For the physicist, the satisfactory explanation of all the main features, apart from the quantitative uncertainty in the details which is an almost inevitable consequence of the complexity of the conditions, must be ranked among the triumphs of science, and constitutes one of the most beautiful applications of the wave theory of light.

The view has long correctly been held that the colour of the sky is due to the presence in the atmosphere of suspended particles, and the explanation was rendered more certain by the experiments of Brücke in 1853 and of Tyndall in 1868 on the colour and polarisation of the diffused light from artificially "clouded" media. The first exact account as to the manner in which the particles produced the effects observed was, however, given by Lord Rayleigh in 1871, in his paper on the light from the sky, its polarisation and colour. It was there shown that the presence in the atmosphere of particles of dimensions small compared with the wave-length of light would give rise to secondary radiations of intensity inversely proportional to the fourth power of the wave-length, and completely polarised in the plane at right angles to the direction of the primary radiation from the sun. In this secondary, diffused radiation, the short waves would therefore greatly preponderate, and the colour seen would be blue or violet, while the long waves would be the more readily transmitted, and the primary radiation seen through such an atmosphere would tend to be orange or red. At the same time, Rayleigh disposed of Clausius's theory that the phenomena were due to the presence in the atmosphere of small—but not small relatively to the wave-length—hollow spherical vesicles reflecting and refracting according to the ordinary laws for extended media. In a much later paper Rayleigh has given good reason for inferring that at least one-third of the scattered light is diffracted from the molecules of the air themselves (see also NATURE, March 10, 1910, p. 49).

One of the merits of Exner's discussion of the subject is the care with which he has followed out the application of Rayleigh's theory in the light of the best recorded observations. These relate to colour, polarisation, extinction coefficients, the general brightness and the distribution of brightness of the sky, &c.; indeed, the whole volume may almost be regarded as an excursus on the Rayleigh theory. In dealing with the more detailed phenomena of Arago's "positive" and "negative" polarisation, and the neutral points of Arago, Babinet, and Brewster, the author follows Soret in attempting a general explanation in which account is taken of the further action of other particles on the light already once diffracted, and of the form of the limited portion of the atmosphere from which the light reaching any individual eye can be received; but the conditions are too complicated to admit of quantitative treatment for exact comparison with observation.

The optical appearances connected with sunrise and sunset are somewhat briefly treated, reference being especially made to Kiessling's monograph "*Untersuchungen über Dämmerungserscheinungen*" for a more complete description of the phenomena. Some account of the observed effects due to the Krakatoa eruption and other similar disturbances is included.

Dr. Exner has followed Prof. Pernter in the careful reproduction of the best recorded and historically interesting observations of the phenomena. He has himself emphasised the impossibility of reproducing the charm of Pernter's work, dependent as it was on the latter's extensive knowledge of the literature of the subject as well as on his critical judgment. This volume will, however, be welcomed both as a fitting completion of the task undertaken by Prof. Pernter and as a valuable survey of the progress which has been made in the interpretation of the phenomena with which it deals.

### THE PHILOSOPHY OF MATHEMATICS.

*Methodologisches und Philosophisches zur Elementar-Mathematik.* By Dr. G. Mannoury. Pp. viii+279. (Haarlem: P. Visser Azn., 1909.) Price 8s. 10d.

THIS work is the outcome of lectures delivered by the author at the University of Amsterdam, and retains in different ways the marks of its origin. Its frequent digressions, its general discursiveness, and its rather sketchy character make it difficult to describe; and many of the topics are so controversial that where one reader will agree with the author, another, equally competent, will entirely dissent. Still, it is an honest and interesting attempt to deal, from the philosophical side, with the fundamental difficulties of mathematics, and as such deserves attention.

The first part contains five chapters dealing respectively with unity and plurality, finite and infinite numbers, the elementary laws of arithmetic applied to whole numbers, the extension of the idea of number, and, finally, the definition of irrationals. The second part is devoted to geometry, and its four chapters discuss respectively mathematical logic; elementary constructions, postulates, and theory of measurement; non-Euclidean geometries; and the notion of space from the standpoint of physiology and psychology.

A few examples must suffice to illustrate the merits and the defects of the author's procedure. Take the question of defining a unit. After pointing out, rightly enough, that there is no such thing as an objective unit directly perceived, he gives as a formal definition (p. 31):—"Units are sensation-complexes (*Empfindungs-Komplexe*), and a plurality (*Vielheit*) consists of mutually related units." Now, if there is one thing that recent mathematics has done, it is surely to clarify and make precise the notion of a unit apart from all elements of sensation. Verbal definition of a unit is a small matter, of course; the thing to be desired is the complete notion. As a matter of fact, everybody does acquire the notion more or less exactly, long before thinking about defining it; and as to the definition, a kindergarten teacher will suc-

ceed where a philosopher will fail. "These are toys; each toy is a unit among the toys"; "You are my class; each of you is a unit of the class"; such examples will convey the meaning of the term "unit" better than any formal definition. At the same time, if we must have a metaphysical example of a unit, the ego seems to be the best, for it cannot be denied, or affirmed to be a plurality, without an intrinsic contradiction in terms. If Jones makes a statement or forms an opinion, however erroneous, it is *his*, and this "he" is an irreducible entity which has a preeminent claim to be called a unit. It may be remarked that Dr. Mannoury expressly objects to this line of argument, apparently on the ground that the idea of the ego is a derivative one; this may be admitted in a sense, as a fact in the development of an individual consciousness, but it does not make the ego derivative, any more than the deciphering of hieroglyphics in recent times affects the date at which they were carved. Is not this one of those cases where psychology is appealed to where it is really irrelevant, the question being one concerning metaphysical data? We must have something *a priori* and undefined in any science; the question is, how few and how fundamental (or elementary) may we assume these data to be?

A more striking example of the same sort of thing is to be found on p. 263, where the author speaks of "the four-dimensional space-time-notion which is to be regarded as an image of the whole group of sensations." It is almost impossible to give any sense to this phraseology, consistent with either popular or mathematical usage. If it merely means that in abstract kinematics in three-dimensional space there are four independent variables ( $x, y, z, t$ ), it is a very unsatisfactory way of stating a simple fact; and it is very doubtful whether kinematics is, properly, an image of sensations, any more than our sensation of the colour of homogeneous light is imaged by its wave-length.

In treating of the elementary laws of arithmetic, the author, in the text, mainly follows those who appeal to the principle of analogy or "permanence"; he does not give a detailed discussion of the elementary operations. The definition of irrationals is Dedekind's, which is wrongly attributed to Dirichlet; there is a brief account of Peano's system of shorthand, and a section on mathematical induction, with quotations from Poincaré, Couturat, and others. Dr. Mannoury is evidently dissatisfied with Poincaré's arguments, but here, as in other cases, he does not bring forward any very definite statements of his own.

In the geometrical section there are several features of interest, and this is the most readable part of the book. A fair account is given of the different types of three-dimensional geometry, of Hilbert's non-Desarguan system, and of metrical geometry based on a movable standard assumed to be rigid. But there is no discussion of a system of definitions, and the only element treated in any detail is the straight line. With regard to the different types of geometry, the author adopts the sensible attitude that it is now, and always will be, impossible to fix on one as the